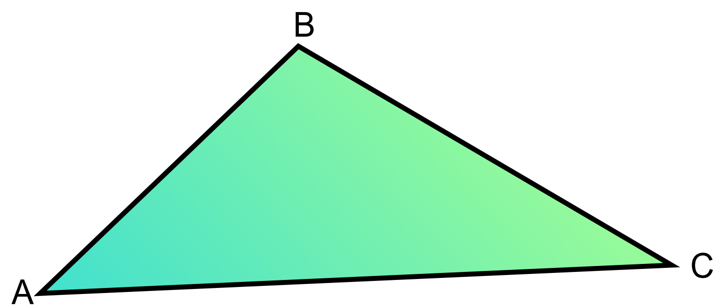
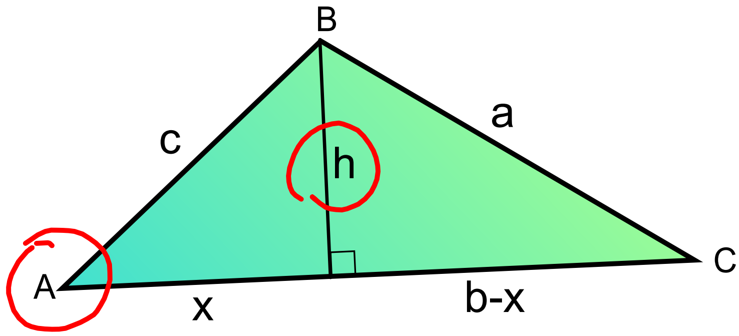
Precalc Warm Up # 11-4

Derive the Law of Cosines:
Come up with an equation relating
a,b,c, and angle A.



Hints: Drop an altitude, and set up a system using the 2 smaller right triangles. Try to come up with an equation that involves only a,b,c and one angle.

Deriving Law of Cosines



$$\cos A = \frac{x}{c}$$

$$x = \boxed{c \cos A}$$

Now find relationships with h in terms of a, b, and c.

$$h^2 = c^2 - x^2$$

$$h^2 = a^2 - (b-x)^2$$

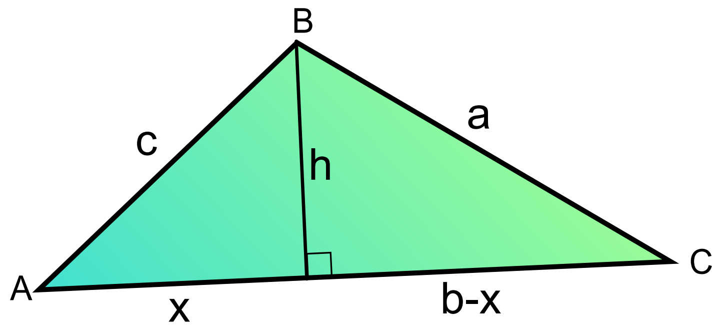
$$h^2 = a^2 - (b^2 - 2bx + x^2)$$

$$c^2 - \cancel{x^2} = a^2 - b^2 + 2bx - \cancel{x^2}$$

$$c^2 = a^2 - b^2 + 2bc \cdot \cos A$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

Deriving Law of Cosines



$$\cos A = \frac{x}{c}$$

$$x = c \cos A$$

Now find relationships with h in terms of a, b, and c.

$$x^2 + h^2 = c^2 \qquad (b-x)^2 + h^2 = a^2$$

$$h^2 = c^2 - x^2 \qquad h^2 = a^2 - (b-x)^2$$

$$c^2 - (c\cos A)^2 = a^2 - (b - c\cos A)^2$$

ta dah!

$$c^2 - c^2\cos^2 A = a^2 - (b^2 - 2bc\cos A + c^2\cos^2 A)$$

$$c^2 = a^2 - b^2 + 2bc\cos A$$

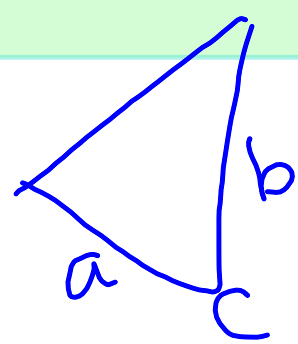
$$= a^2$$

Law Of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

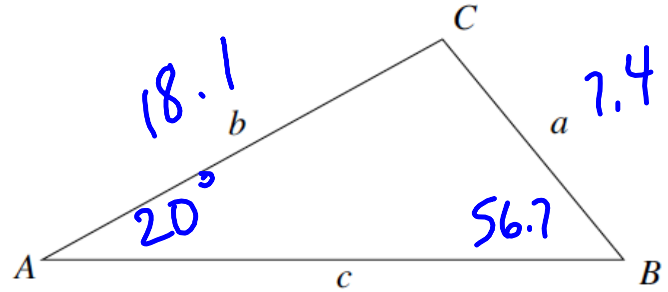
$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXERCISES 9.5.2 p. 297

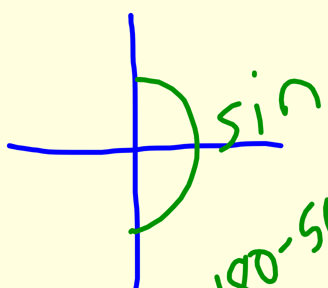
Find the two solutions to these triangles which are defined using the standard labelling:

	a cm	b cm	A
1.	7.4	18.1	20°
2.	13.3	19.5	14°
3.	13.5	17	28°



$$\frac{7.4}{\sin 20^\circ} = \frac{18.1}{\sin B}$$

$$m\angle B = \sin^{-1}\left(\frac{\sin 20^\circ \cdot 18.1}{7.4}\right)$$

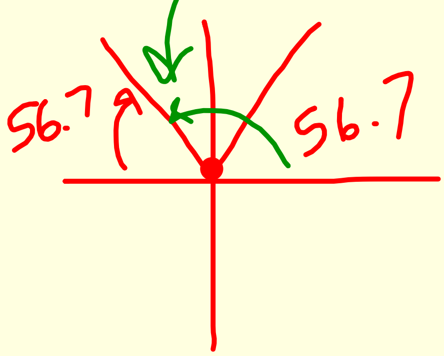


$m\angle B \approx 56.78°$

$m\angle C \approx 103.22°$

$c \approx 21.06$ cm

180-56.7



2nd △

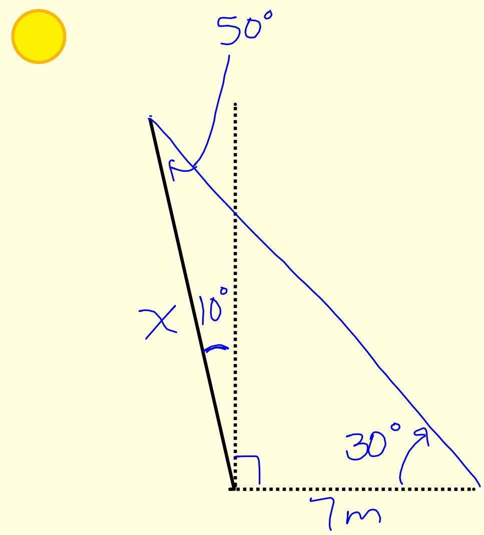
$m\angle B \approx 123.22°$

$m\angle C \approx 36.78°$

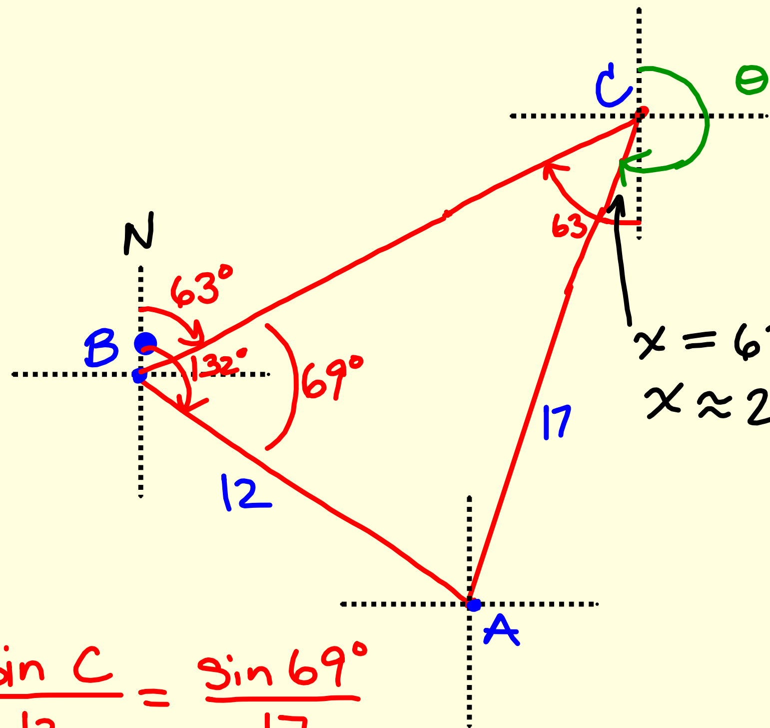
$c \approx 12.95$

p. 298

2. A pole is slanting towards the sun and is making an angle of 10° to the vertical. It casts a shadow 7 metres long along the horizontal ground. The angle of elevation of the top of the pole to the tip of its shadow is 30°. Find the length of the pole, giving your answer to 2 d.p.



4. Town A is 12 km from town B and its bearing is 132˚T from B. Town C is 17 km from A and its bearing is 063˚T from B. Find the bearing of A from C.



$\theta = 180 + x \approx 201°47'$

$\boxed{201°47'}$

$x = 63 - C$

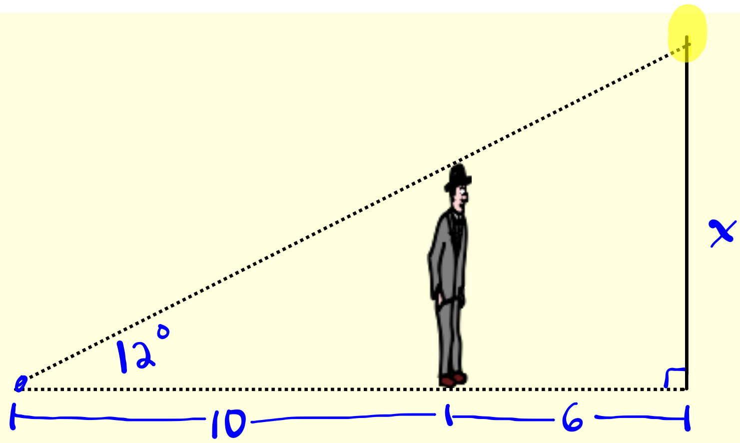
$x \approx 21.8°$

$$\frac{\sin C}{12} = \frac{\sin 69°}{17}$$

$$C = \sin^{-1}\left(\frac{12 \sin 69°}{17}\right) \approx 41.2° \quad \text{STO} \rightarrow C$$

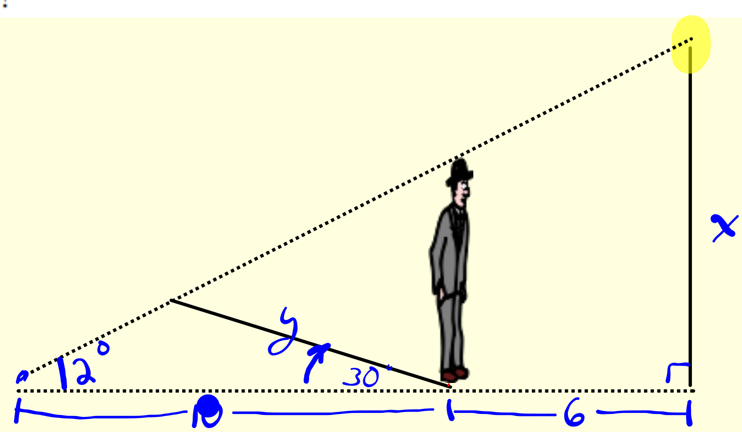
6. (a) A man standing 6 metres away from a lamp post casts a shadow 10 metres long on a horizontal ground. The angle of elevation from the tip of the shadow to the lamp light is 12°. How high is the lamp light?

(b) If the shadow is cast onto a road sloping at 30° upwards, how long would the shadow be if the man is standing at the foot of the sloping road and 6 metres from the lamp post?



6. (a) A man standing 6 metres away from a lamp post casts a shadow10 metres long on a horizontal ground. The angle of elevation from the tip of the shadow to the lamp light is 12˚. How high is the lamp light?

(b) If the shadow is cast onto a road sloping at 30˚ upwards, how long would the shadow be if the man is standing at the foot of the sloping road and 6 metres from the lamp post?



8. The lookout on a ship sailing due East at 25 km/h observes a reef N62˚E at a distance of 30 km.

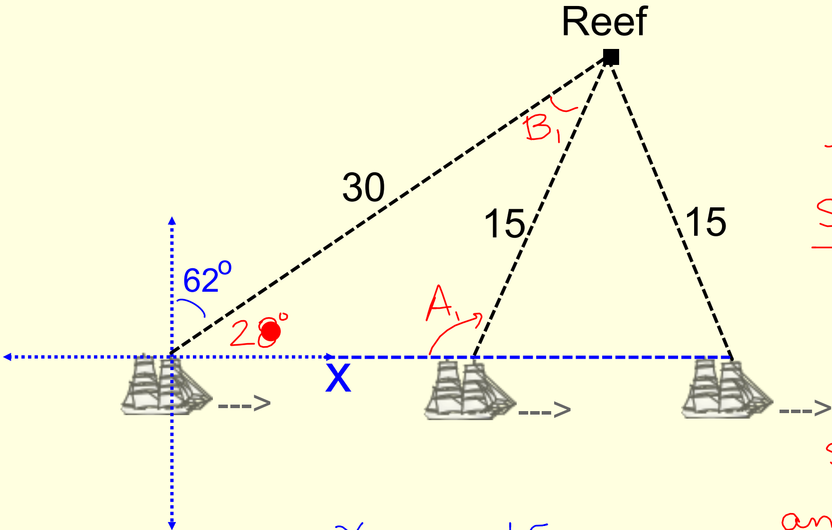
(a) How long will it be before the ship is 15 km from the reef, assuming that it continues on its easterly course.

(b) How long is it before it is again 15 km from the reef?

(c) What is the closest that the ship will get to the reef?

a) $x\ \cancel{km} \cdot \frac{1\ hr.}{25\ \cancel{km}}$

$\frac{x}{25}$ hrs.



finding x:

$$\frac{\sin A}{30} = \frac{\sin 28^\circ}{15}$$

$$A = \sin^{-1}(2 \sin 28^\circ)$$

$$A \approx 69.9^\circ$$

so $A_1 \approx 110.1^\circ$

and $B_1 \approx 41.9^\circ$ STO→B

$$\frac{x}{\sin B_1} = \frac{15}{\sin 28^\circ}$$

$$x = \frac{15 \sin B_1}{\sin 28^\circ} \approx 21.3 \text{ km.}$$

Now ÷ 25

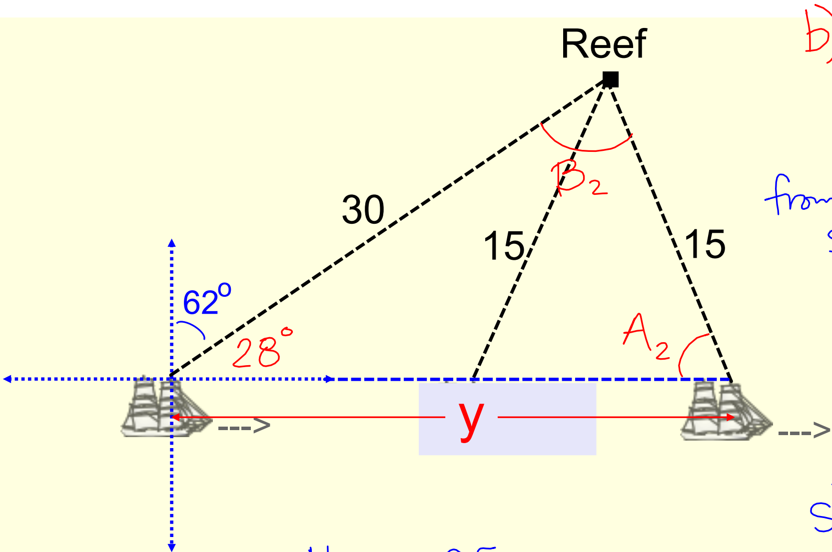
It will take 0.85 hrs or ≈ 51 minutes.

8. The lookout on a ship sailing due East at 25 km/h observes a reef N62°E at a distance of 30 km.

(a) How long will it be before the ship is 15 km from the reef, assuming that it continues on its easterly course.

(b) How long is it before it is again 15 km from the reef?

(c) What is the closest that the ship will get to the reef?



b) $\frac{y}{25}$ hours

finding y:

from last slide } $A_2 = \sin^{-1}(2\sin 28°)$

$A_2 \approx 69.9°$

$B_2 \approx 82.1°$ STO → B

$$\frac{y}{\sin B} = \frac{15}{\sin 28°}$$

$$y = \frac{15 \sin B}{\sin 28°}$$

$y \approx 31.6$ km

Now ÷ 25

It took ≈ 1.27 hrs

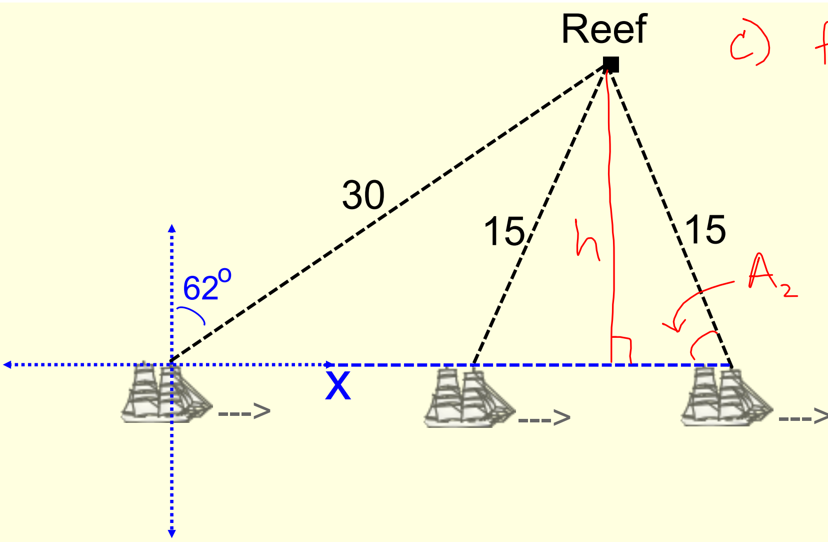
or ≈ 1 hr 16 min.

8. The lookout on a ship sailing due East at 25 km/h observes a reef N62˚E at a distance of 30 km.

(a) How long will it be before the ship is 15 km from the reef, assuming that it continues on its easterly course.

(b) How long is it before it is again 15 km from the reef?

(c) What is the closest that the ship will get to the reef?



c) from before:

$A_2 = \sin^{-1}(2 \sin 28°)$

$A_2 \approx 69.9°$

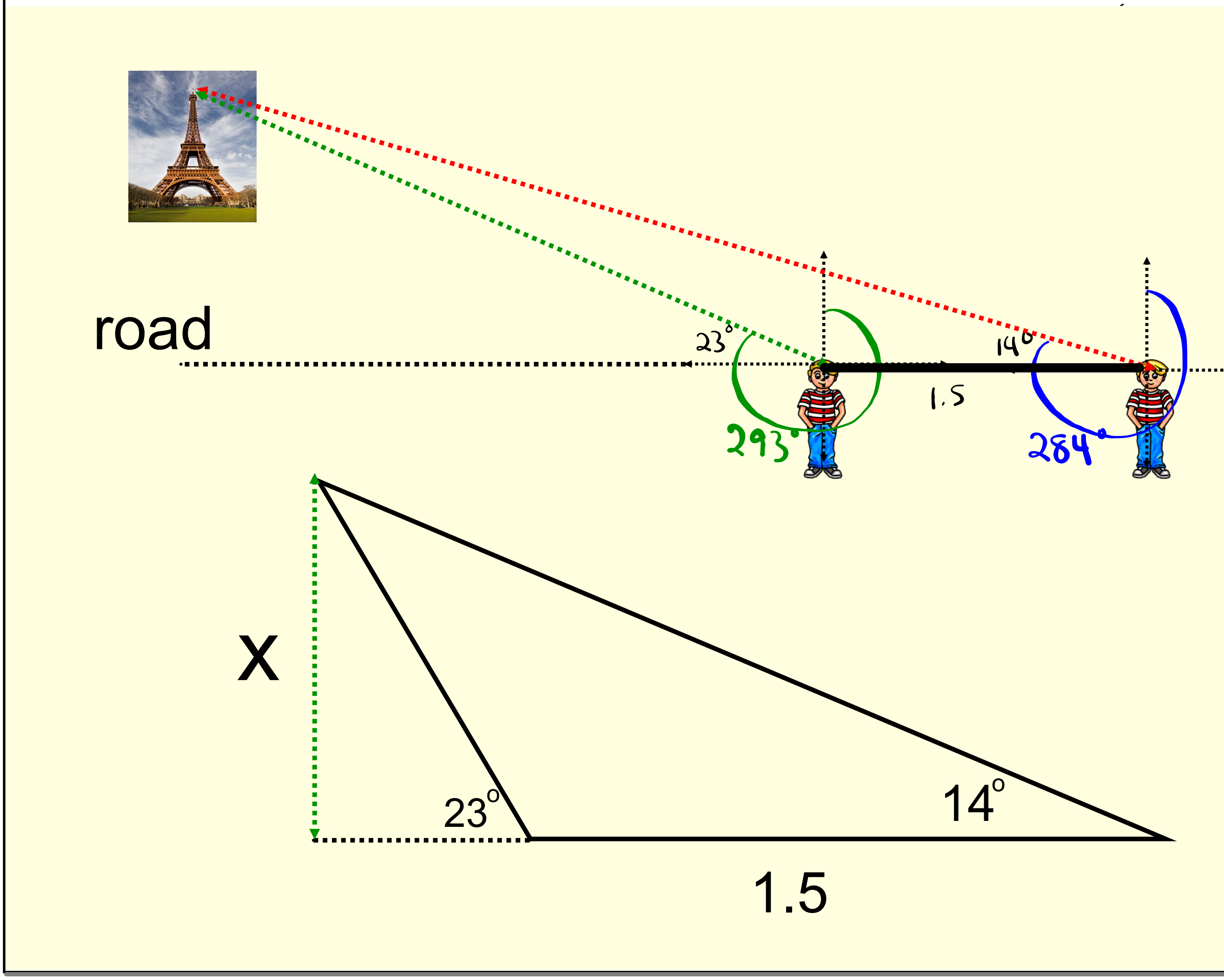
STO → A

$\sin A_2 = \frac{h}{15}$

$h = 15 \sin A$

$h \approx 14.1$ km

10. A boy walking along a straight road notices the top of a tower at a bearing of 284°T. After walking a further 1.5 km he notices that the top of the tower is at a bearing of 293°T. How far from the road is the tower?



Law Of Cosines:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Solve the formula for angle A:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$-b^2 \quad -c^2$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \frac{-\cancel{2bc} \cdot \cos A}{-\cancel{2bc}}$$

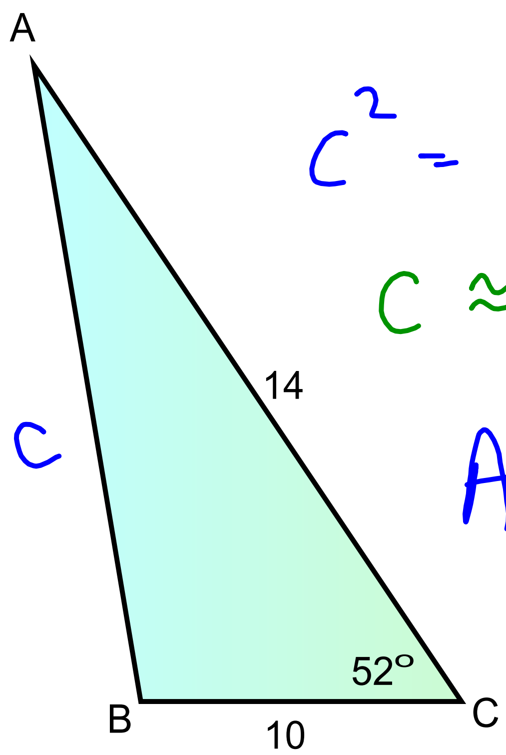
$$\cos A = \frac{-a^2 + b^2 + c^2}{2bc}$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

Solve the SAS triangle. Use only Law of Cosines.
Can there be 2 sets of answers? Can we get no solution?



$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

$$c \approx 11.1$$

$$A = \cos^{-1}\left(\frac{(b^2 + c^2 - a^2)}{(2bc)}\right)$$

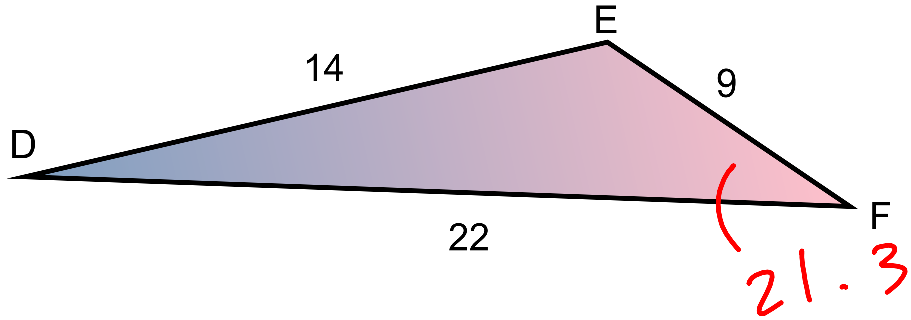
$$A \approx 45.5^\circ$$

$$B \approx 180^\circ - 52^\circ - 45.5^\circ$$

note: Don't assume the triangles are drawn to scale!

$$B \approx 82.5^\circ$$

Solve the SSS triangle. After you find one angle you can switch to Law of Sines for the rest... with caution!

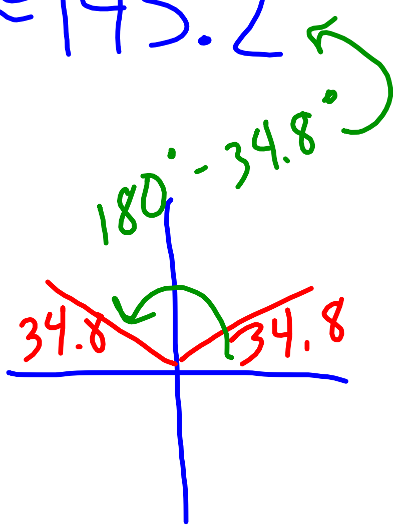


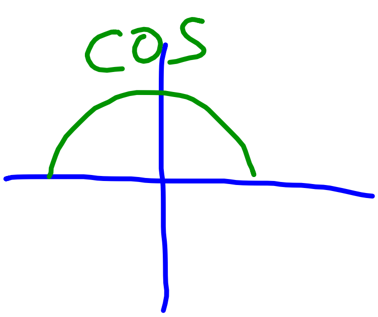
Does it matter which angle you find first?

Find E first:

$$E = \cos^{-1}\left(\frac{d^2+f^2-e^2}{2df}\right)$$

$$E \approx 145.2^\circ$$





Find F first, then E with Law of Sines:

$$F = \cos^{-1}\left(\frac{e^2+d^2-f^2}{2ed}\right)$$

$$F \approx 21.3^\circ$$

STO → F

$$\frac{\sin E}{22} = \frac{\sin F}{14}$$

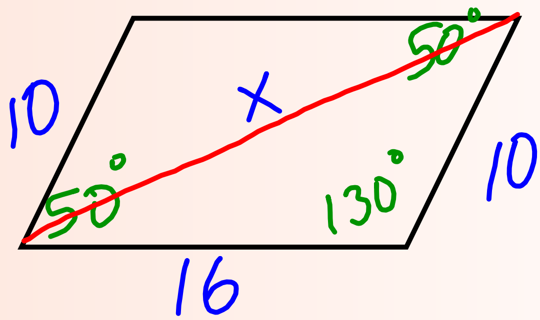
$$E = \sin^{-1}\left(\frac{22\sin F}{14}\right)$$

$$E \approx 34.8^\circ$$

groups:

Two sides of a parallelogram are 16 and 10. If their included angle is 50°, find the length of the longest diagonal.

$(n-2)180 = \text{sum int } \angle\text{'s}$

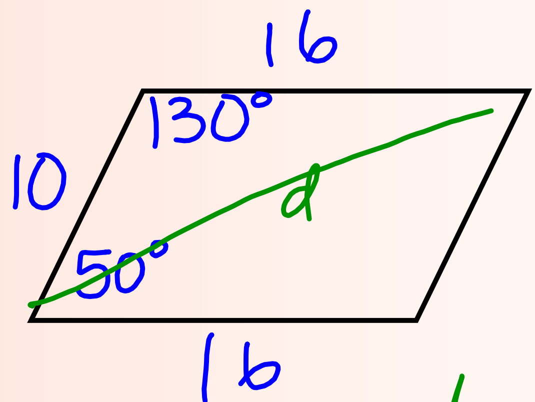


$$x^2 = 16^2 + 10^2 - 2(16)(10) \cdot \cos 130°$$

$$x \approx 23.7$$

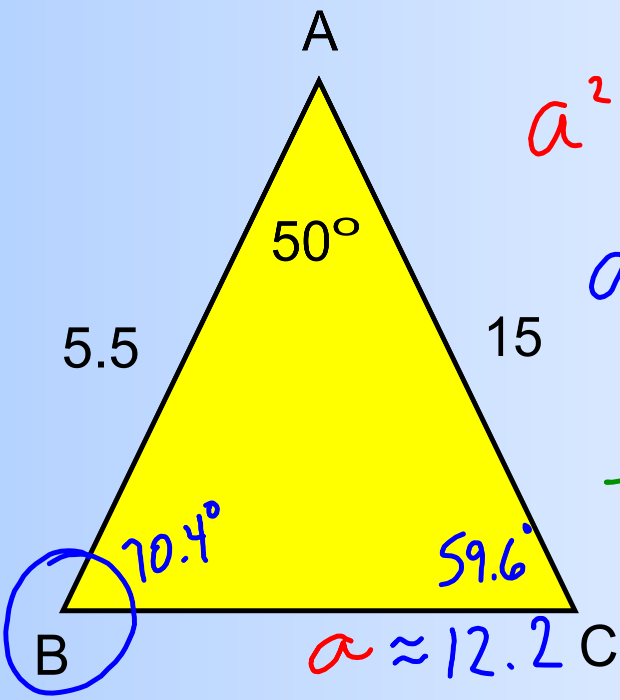
groups:

Two sides of a parallelogram are 16 and 10. If their included angle is 50°, find the length of the longest diagonal.



$d \approx 23.7$

Find the measure of the largest angle in the triangle.



$$a^2 = 15^2 + 5.5^2 - 2(15)(5.5)\cdot\cos 50$$

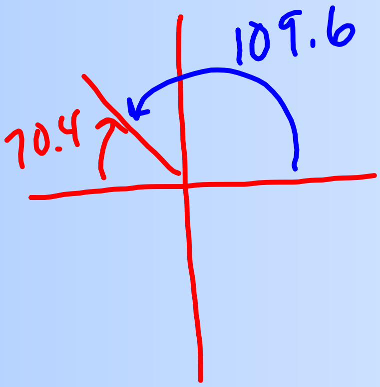
$$a \approx 12.2$$

$$\frac{12.2}{\sin 50^\circ} = \frac{15}{\sin B}$$

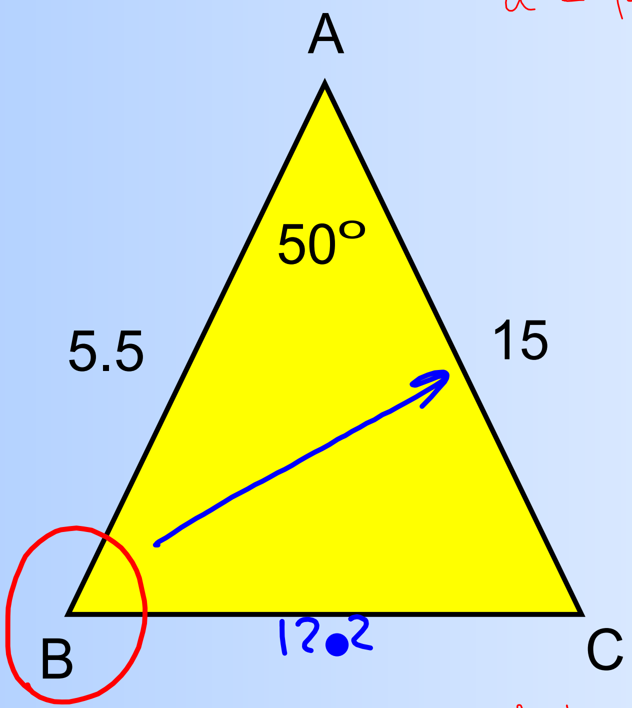
$$B = \sin^{-1}\left(\frac{15\cdot\sin 50^\circ}{12.2}\right)$$

can't be $B \approx \cancel{70.4^\circ}$

$$C \approx 59.6$$



Find the measure of the largest angle in the triangle.



$a^2 = 15^2 + 5.5^2 - 2(15)(5.5)\cos 50°$

$a \approx 12.2$ STO → A

$m\angle B \approx 70.4$

$m\angle C \approx 59.6$

$$\frac{\sin B}{15} = \frac{\sin 50}{a}$$

$$\sin^{-1}\left(\frac{15\sin 50}{a}\right)$$

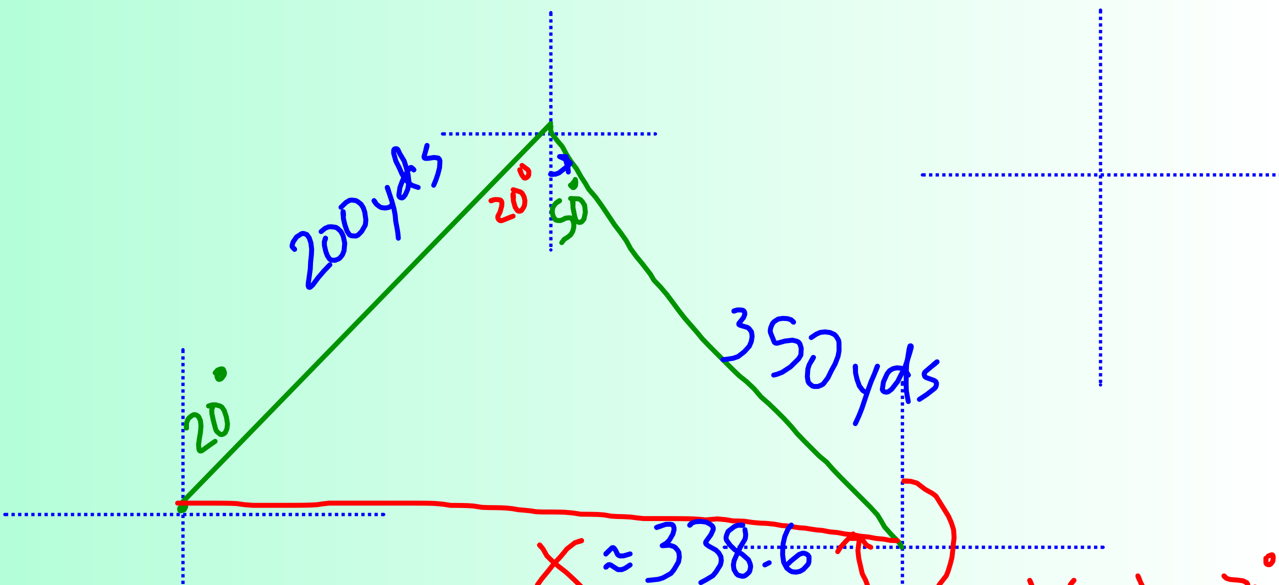
$B \approx 70.4$

But $\angle C$ should be the smallest $\angle$.

so $\angle B$ must be obtuse!

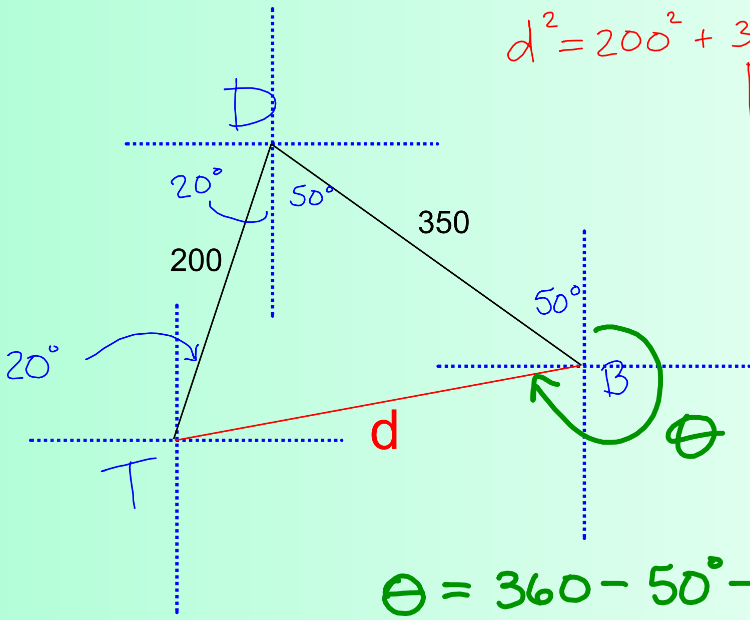
* Always make sure everything makes sense when using Law of Sines!

Casey hits his golf ball 200 yards at a 20°T bearing. He then hits it 350 yards S 50°E. How far (nearest yard) is the ball from his starting point? If he has to return to the tee box, what bearing (True, nearest 10th of degree) must he take from the last position of the ball?



$$x^2 = 200^2 + 350^2 - 2(200)(350)\cdot\cos 70^\circ$$

Casey hits his golf ball 200 yards at a 20°T bearing. He then hits it 350 yards S 50°E. How far (nearest yard) is the ball from his starting point? If he has to return to the tee box, what bearing (True, nearest 10th of degree) must he take from the last position of the ball?



$d^2 = 200^2 + 350^2 - 2(200)(350)\cos 70°$

$\boxed{d \approx 338.6 \text{ yds.}}$

STO → d

$\frac{\sin B}{200} = \frac{\sin 70°}{d}$

$B = \sin^{-1}\left(\frac{200 \sin 70°}{d}\right)$

$B \approx 33.7°$

STO → B

$\theta = 360 - 50° - B$

$\theta \approx 276.3°$

$\boxed{\approx 276.3° \text{ T}}$

HW: SL book, p. 302 #2, 15,

p. 304 #1 - 6 at the top

Group Test next week
(solve triangles)

Individual Unit test: next week

SL book: 9.2, 9.4, 9.5

Test covers:

Law of Sines
Law of Cosines
Area of non-right triangles
Angles of Elevation and Depression
Bearing
Solving Triangles